



# A numerical study on the transient heat transfer from a sphere at high Reynolds and Peclet numbers

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## Abstract

The transient heat transfer rate from a particle depends on the flow field as well as the temperature field developed around the particle. The parameters that determine these two fields are the Reynolds number and the Peclet number. A numerical study has been performed in order to determine the transient heat transfer from a spherical particle in terms of both of these parameters. The governing equations of the problem are made dimensionless and are solved by using the stream function-vorticity formulation. The solution of the equations is achieved by using a stretched coordinate system and a tri-diagonal matrix algorithm. Good agreement of the numerical results was observed with previous studies on drag coefficients as well as with analytical and asymptotic expressions derived in the past. The results show a strong dependence of the rate of heat transfer on the Reynolds number, when the Peclet and Reynolds numbers are higher than one. © 1999 Elsevier Science Ltd. All rights reserved.

*Keywords:* Transient; Particles; Drops; High Reynolds Numbers; Numerical

## 1. Introduction

The problem of heat transfer from a sphere is a classical one and has been the subject of several investigations in the past, starting with the pioneering work of Fourier [1] whose primary interest was the cooling of the planets. Because the problem is of prime scientific importance and, in addition, has many engineering applications, it has been revisited many times. Most of the theoretical work on the subject pertains to steady-state solutions for the heat transfer from an isothermal sphere. Acrivos and Taylor [2] used a singular perturbation analysis and a Stokesian velocity distribution

around the sphere, to derive their well-known solution for the steady-state heat transfer from a sphere at small but finite Peclet numbers. Brenner [3] very soon extended this solution to the case of non-spherical particles. Analytical and experimental results (in the form of correlations) on the steady-state heat transfer coefficients are abundant in heat transfer and transport properties textbooks [4].

A solution to the problem of transient heat transfer from a sphere at creeping flow ( $Re = 0$ ) appears in Carslaw and Jaeger [5]. An analytical solution on the unsteady heat transfer from a sphere at low Reynolds numbers under steady velocity conditions was developed by Choudhury and Drake [6]. They used the steady-state velocity field developed by Proudman and Pearson [7], which is applicable at Reynolds numbers small in comparison to 1. Feng and Michaelides [8]

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**Nomenclature**

$a$	radius	$x, y$	coordinates in the stretched computational domain
$c$	specific heat capacity		
$E$	operator defined in Eq. (6)		
erf	error function		
$g$	gravitational acceleration	<i>Greek symbols</i>	
$H(t)$	heavyside function	$\zeta$	vorticity
$k$	thermal conductivity	$\theta$	azimuthal coordinate
$Nu$	Nusselt number	$\Theta$	temperature
$p$	pressure	$\nu$	kinematic viscosity
$r$	radial direction	$\rho$	density
$Pe$	Peclet number based on radius	$\tau_s$	characteristic time for conduction
$Pe_D$	Peclet number based on diameter	$\Psi$	stream function
$Re$	Reynolds number based on the radius	$\Omega$	vector related to vorticity
$Re_D$	Reynolds number based on the diameter	<i>Subscript</i>	
$t$	time	f	pertains to friction
$U$	velocity	p	pertains to pressure
$v$	velocity	S	pertains to the surface of the sphere
		$\infty$	pertains to conditions far from the sphere

also derived analytically an expression for the heat transfer from a small sphere at low Peclet numbers assuming a Stokesian velocity distribution around the sphere and pointed out the effects of the history terms in the resulting expressions. Abramzon and Elata [9] conducted a numerical study for the transient heat transfer from a rigid sphere at a wide range of Peclet numbers, also assuming a Stokesian velocity distribution around the sphere. Their study is valid at very low Reynolds numbers. Because of this, their numerical results are restricted to extremely low Prandtl numbers (when  $Pe \gg 1$ ), where there are very few practical applications. It must also be pointed out that, because of the importance of the problem in evaporation processes, several studies have been conducted on the heat and mass transfer from one or more evaporating droplets at steady flow conditions [10] as well as on the interactions of these droplets [11].

In most practical applications of heat or mass transfer from rigid spheres, the sizes and properties of the particles are such that both  $Pe$  and  $Re$  are finite. Because of this, results derived under the explicit or implicit assumption that the values of either  $Re$  or  $Pe$  are very low, may not be valid in practice. Furthermore, it is apparent that the general problem of transient heat and mass transfer from particles encompasses two independent dimensionless parameters, the Peclet number and the Reynolds number. The two numbers account for the temperature and velocity fields, which determine the heat transfer process. Therefore, any accurate study on the subject must

include both of these numbers explicitly. This task is accomplished in this manuscript: we obtain the velocity field around the spherical particle by numerically solving the Navier–Stokes equations for the range of Reynolds numbers for the sphere 0 to 2000. Then we use this information to solve the energy equation and to derive the rate heat transfer for a sphere for the range of Peclet numbers from 0 to 1000.

**2. Governing equations**

The problem considered may be described without any loss of generality as a unidirectional flow with velocity  $U_\infty$  in the  $-e_2$  direction past a sphere of radius  $a$ . In this case, the momentum equation for the velocity field developed in the vicinity of the sphere, may be written as follows:

$$\vec{v} \cdot \vec{\nabla} \vec{v} = -\frac{1}{\rho} \vec{\nabla} p + \vec{g} + \nu \nabla^2 \vec{v} \quad (1)$$

and the continuity equation is

$$\vec{\nabla} \cdot \vec{v} = 0 \quad (2)$$

where  $\rho$  is the density of the fluid,  $\nu$  is the dynamic viscosity of the fluid, the vector  $g$  is the gravitational acceleration,  $p$  is the pressure and the vector  $v$  is the velocity of flow. Associated with the velocity field are the stream function,  $\Psi$  and the vorticity vector  $\zeta$ .

We first make the equations dimensionless by using

the following expressions:

$$r = \frac{r'}{a}, \quad \Psi = \frac{\Psi'}{U_\infty a^2}, \quad v = \frac{v'}{U_\infty}, \quad t = \frac{t' U_\infty}{a} \quad (3)$$

where a prime denotes the dimensional value. Then the equations are written in spherical coordinates and in the stream function–vorticity formulation. Since the problem is axisymmetric, the equations for the vorticity vector  $\zeta$  may be written as follows:

$$E^2 \Psi = \Omega = \frac{\zeta_3}{r \sin \theta} \quad (4)$$

and

$$\begin{aligned} \sin(\theta) \left[ \frac{\partial \Psi}{\partial r} \frac{\partial}{\partial \theta} \left( \frac{\Omega}{r^2 \sin^2 \theta} \right) - \frac{\partial \Psi}{\partial \theta} \frac{\partial}{\partial r} \left( \frac{\Omega}{r^2 \sin^2 \theta} \right) \right] \\ = \frac{1}{Re} E^2 \Omega \end{aligned} \quad (5)$$

The operator  $E$  is defined as follows:

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right) \quad (6)$$

In the last equation,  $Re$  is the Reynolds number, defined in terms of the radius as follows:

$$Re = \frac{a U_\infty}{\nu} \quad (7)$$

The latter is equal to one-half the particle Reynolds number  $Re_D$ , which is defined in terms of the diameter of the sphere.

The boundary conditions for this problem are:

1. no velocity slip at the surface of sphere,
2. symmetry with respect to the  $x_2$ -axis, and
3. unit velocity component in the direction of the vector  $e_2$  at infinity.

Thus, the three boundary conditions may be written as follows in terms of  $\Psi$  and  $\Omega$ :

$$\Psi = \frac{\partial \Psi}{\partial r} = 0 \quad \text{at } r = 1$$

$$\Psi = \Omega = 0 \quad \text{on } \theta = 0, \pi$$

$$\Psi \rightarrow \frac{1}{2} r^2 \sin^2 \theta \quad \text{as } r \rightarrow \infty \quad (8)$$

### 2.1. Energy equation

We consider the case of the sphere experiencing a step temperature change at time  $t = 0$  from  $\Theta_\infty$  to  $\Theta_s$ . For simplicity, we assume the ambient fluid tempera-

ture of fluid far from the sphere to be  $\Theta_\infty$  (that is, initially the fluid and the sphere are in equilibrium). We define the following dimensionless temperature:

$$\Theta = \frac{\Theta' - \Theta_\infty}{\Theta_s - \Theta_\infty} \quad (9)$$

and, hence, we write the dimensionless unsteady heat transfer equation as follows:

$$Pe \frac{\partial \Theta}{\partial t} + Pe \vec{v} \cdot \vec{\nabla} \Theta = \nabla^2 \Theta \quad (10)$$

where the Peclet number  $Pe$  is defined in terms of the radius of the sphere as follows:

$$Pe = \frac{\rho c a U_\infty}{k} \quad (11)$$

$c$  is the specific heat of the fluid, and  $k$  its conductivity.

We substitute for the velocity vector in terms of the stream function and obtain the following dimensionless form of the energy equation:

$$\begin{aligned} Pe \frac{\partial \Theta}{\partial t} + Pe \left[ \frac{\partial \Psi}{\partial r} \frac{\partial}{\partial \theta} \left( \frac{\Theta}{r \sin \theta} \right) - \frac{\partial \Psi}{\partial \theta} \frac{\partial}{\partial r} \left( \frac{\Theta}{r \sin \theta} \right) \right] \\ = \Delta^2 \Theta \end{aligned} \quad (12)$$

The initial condition for the energy equation is

$$\Theta = H(t) \quad \text{at } r = 1 \quad (13a)$$

where  $H(t)$  is the unit step function. The boundary conditions of this equation are

$$\Theta = 1 \quad \text{at } r = 1, t > 0; \quad \text{and} \quad \Theta \rightarrow 0 \quad \text{as } r \rightarrow \infty \quad (13b)$$

Eqs. (5) and (12) are a system of non-linear equations, which will be solved numerically. Eq. (5) will be solved first and the results for the stream function (or velocity) field will be used in the solution of Eq. (12), with the boundary conditions as stipulated above.

### 3. Numerical method

Because the problem is symmetric, the numerical computation is performed in a semicircular  $(r, \theta)$  domain. Grid sensitivity analysis proved that an accurate solution is obtained with a grid equal to 130 radii when either  $Re$  or  $Pe$  is less than 10 and a grid extending to 90 radii when both  $Re$  and  $Pe$  are higher than 10. In order to achieve a more dense grid near the surface of the sphere, a logarithmic coordinate stretch ( $x = \theta, y = \log r$ ) was performed in the radial direc-

It is observed that the numerical value of  $\Psi$  is in the range from 0 to  $\exp(2y_{\text{End}})$  where  $y_{\text{End}}$  is the maximum coordinate in the computational domain (90–130). This is a very large range and normally would result in high numerical errors. For this reason, the stream function  $\Psi(x,y)$  is separated into two parts: a steady potential part that accounts for the large variation in  $\Psi$  and a viscous correction  $\psi$ :

$$\Psi(x, y) = \Psi_p(x, y) + \psi(x, y) \quad (14)$$

where  $\Psi_p(x,y)$  is the well-known solution of the potential flow problem,  $E^2\Psi_p = 0$ , which is as follows:

$$\Psi_p(x, y) = \frac{1}{2}(e^{2y} - e^{-y}) \sin^2(x) \quad (15)$$

Therefore, the velocity field is obtained as the solution of the following equations:

$$\left[ \frac{\partial}{\partial y^2} - \frac{\partial}{\partial y} + \frac{\partial}{\partial x^2} - \cot x \frac{\partial}{\partial x} \right] \psi = r^2 \Omega \quad (16)$$

and

$$\begin{aligned} & \frac{\partial}{\partial x} \left( \frac{\Omega}{e^{2y} \sin^2 x} \frac{\partial \psi}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{\Omega}{e^{2y} \sin^2 x} \frac{\partial \psi}{\partial x} \right) \\ &= \frac{1}{Re e^y \sin x} \left[ \frac{\partial}{\partial y^2} - \frac{\partial}{\partial y} + \frac{\partial}{\partial x^2} - \cot x \frac{\partial}{\partial x} \right] \Omega \end{aligned} \quad (17)$$

with boundary conditions:

$$\psi = 0 \quad \text{at } y = 0; \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = y_{\text{End}}; \quad (18a)$$

and

$$\Omega = \frac{\partial^2 \psi}{\partial y^2} + \frac{3}{2} \sin^2(x) \quad \text{at } y = 0 \quad (\text{or } r = 1); \quad (18b)$$

In a similar way, the governing equation for heat transfer in the stretched coordinate system becomes

$$\begin{aligned} & Pe \frac{e^y}{\sin(x)} \frac{\partial \Theta}{\partial t} + \frac{Pe}{e^{2y} \sin^2(x)} \left[ \frac{\partial}{\partial x} \left( T \frac{\partial \Psi}{\partial y} \right) - \frac{\partial}{\partial y} \left( T \frac{\partial \Psi}{\partial x} \right) \right] \\ &= \frac{1}{e^y \sin(x)} \left[ \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial \Theta}{\partial y} + \frac{\partial^2 \Theta}{\partial x^2} + \cot(x) \frac{\partial \Theta}{\partial x} \right] \end{aligned} \quad (19)$$

and the boundary conditions are

$$\Theta = 1 \quad \text{at } y = 0; \quad \Theta \approx 0 \quad \text{at } y = y_{\text{End}} \quad (20)$$

For the solution of these differential equations we have employed the control-volume formulation [12]. In order to account for the fact that at relatively high  $Re$

or  $Pe$  the solution does not easily converge, due to the added weight of the advective term, an upwind scheme [13] was used. For the solution of the transient energy equation, we use a fully-explicit scheme. The upwind scheme is also applied here for the advection term. The discretized stream-vorticity equations and energy equation are solved by a line-by-line TDMA (TriDiagonal-Matrix Algorithm). It must be pointed out that the algebraic equations for  $\psi$  and  $\Omega$  were solved simultaneously and that the boundary condition for  $\Omega$  at  $y = 0$  is updated after every iteration. The convergence criterion for the iterations is chosen, such that the largest relative change of any quantity,  $\phi$ , between two consecutive iterations is less than a parameter,  $Er$ , which was equal to  $10^{-6}$ .

$$\frac{|\phi^{(n+1)} - \phi^{(n)}|}{\max(|\phi^{(n)}|, 1)} \leq Er \quad (21)$$

The numerical procedure is robust and yields accurate numerical results for very high Reynolds numbers. We tested it for up to  $Re = 10,000$  and encountered no difficulties in convergence.

## 4. Results, validation and discussion

### 4.1. Drag coefficient

In order to validate the numerical results, we first calculated the drag coefficient for an isothermal sphere and compared the results with those of Dennis et al. [14] and LeClair et al. [15]. The calculations were made using a grid of  $100 \times 120$  and the value of  $Er$  was taken as  $10^{-6}$ . The Reynolds number of the particle, based on the particle diameter ( $Re_D = 2\alpha U_\infty / \nu = 2Re$ ) was the parameter. The results of the calculations for the drag coefficient and the comparison with the known results appear in Table 1.

It is evident that the values of the drag coefficient obtained from this study are in very good agreement (less than 2% deviation) with the previous studies as well as with Stokes' expression  $C_D \rightarrow 24/Re_D$  as  $Re_D \rightarrow 0$ . It must be pointed out that all the experimental studies on the subject have higher than 2% uncertainty. Because of this, it is meaningless to assert which one of the numerical studies compares better with experimental data. We are confident on the accuracy of our data, because they result from a more refined numerical method and a denser grid than those used in the previous studies.

Using this method we were able to extend our results to very high  $Re$  and  $Re_D$  (up to 2000 and 4000, respectively). We separated the dimensionless drag

Table 1  
Comparison of the results for  $C_D$  with other numerical studies

Reynolds number, $Re_D$	LeClair et al. [15]	Dennis et al. [14]	Present results
0.01	244.08	244.20	241.66
1	27.32	27.44	26.98
5	7.03	7.21	7.04
20	2.712	2.73	2.682
40	1.86	1.808	1.77

force on the particle into two parts: one due to the pressure gradient, and the other due to friction;

$$F = F_p + F_f \quad (22)$$

The pressure and friction components of the drag force and the drag coefficient may be calculated [16] from the following expressions:

$$F_p = -\pi \int_0^\pi \sin(2\theta) \left[ \int_0^\theta \left( \frac{\partial \zeta}{\partial r} + \frac{\zeta}{r} \right) \Big|_{r=1} d\theta' \right] d\theta \quad (23)$$

and

$$F_f = -2\pi \int_0^\pi \zeta|_{r=1} \sin^2 \theta d\theta \quad (24)$$

The results for the two corresponding drag coefficients and their sum,  $C_D$  are given in Table 2. It is obvious that these results agree well with the asymptotic analytical results:  $C_D \rightarrow 24/Re_D$  as  $Re_D \rightarrow 0$ ,  $C_f \rightarrow 16/Re_D$  as  $Re_D \rightarrow 0$ ,  $C_p \rightarrow 8/Re_D$  as  $Re_D \rightarrow 0$  and  $C_f \ll C_p$  as  $Re_D \rightarrow \infty$ . The results, up to  $Re_D = 4000$  also agree very well with the correlations given by White [17] and the recommended correlations by Clift et al. [18]. Fig. 1 depicts the results of this study with the values obtained from the two correlations. It is obvious that

Table 2  
Drag coefficients for an isothermal sphere

Reynolds number, $Re_D$	$C_f$	$C_p$	$C_D$
0.1	163.36	78.298	241.658
0.2	82.850	39.706	122.556
0.5	34.484	16.536	51.020
1	18.216	8.756	26.972
5	4.694	2.346	7.040
20	1.694	0.988	2.682
40	1.055	0.7152	1.7702
100	0.5714	0.5234	1.0948
500	0.1926	0.4072	0.5998
1000	0.1206	0.4000	0.5206
2000	0.07514	0.38684	0.46198
4000	0.04588	0.32476	0.37064

there is good agreement, a fact that validates the numerical results of this study to approximately  $Re_D = 4000$ . It must be pointed out, however, that for higher values of  $Re_D$ , the numerical results underpredict the correlations (as shown with the last point of Fig. 1). This problem is due to vortex shedding from the particle. At high  $Re_D$  the effect of the shed vortices on the flow field developed around the particle becomes significant outside the computational domain and, therefore cannot be captured by any numerical scheme, without a corresponding extension of the computational domain.

Some of the intermediate results on the vorticity field created around the sphere are given in Fig. 2a and b. The two figures show the vorticity field for values of  $Re_D$  40 and 1000. It is obvious in the second case that a well-formed wake exists downstream the sphere, whose main characteristics are well captured by the numerical scheme employed here.

#### 4.2. Temperature field and Nusselt numbers

The heat transfer from the sphere to the fluid may be expressed by the instantaneous Nusselt number,  $Nu(t)$ . The local Nusselt number is given by the equation

$$Nu_x = \frac{\partial \Theta}{\partial y} \Big|_{y=0} \quad (25)$$

and the average Nusselt number is the integral of this quantity, taken around the sphere. The average Nusselt number was computed in this study, utilizing the previously obtained results for the velocity field. We validated the results of our numerical solution by comparing them to steady-state data and correlations. The Whitaker correlation [19] was used for this purpose. The results of the comparison are depicted in Fig. 3, for three Prandtl numbers 0.1, 1 and 10. The predictions of the numerical method are well within the uncertainty of the correlation, which is of the order of 20%. Excellent agreement was also observed with the experimental data and the resulting correlations by Yuge [20].

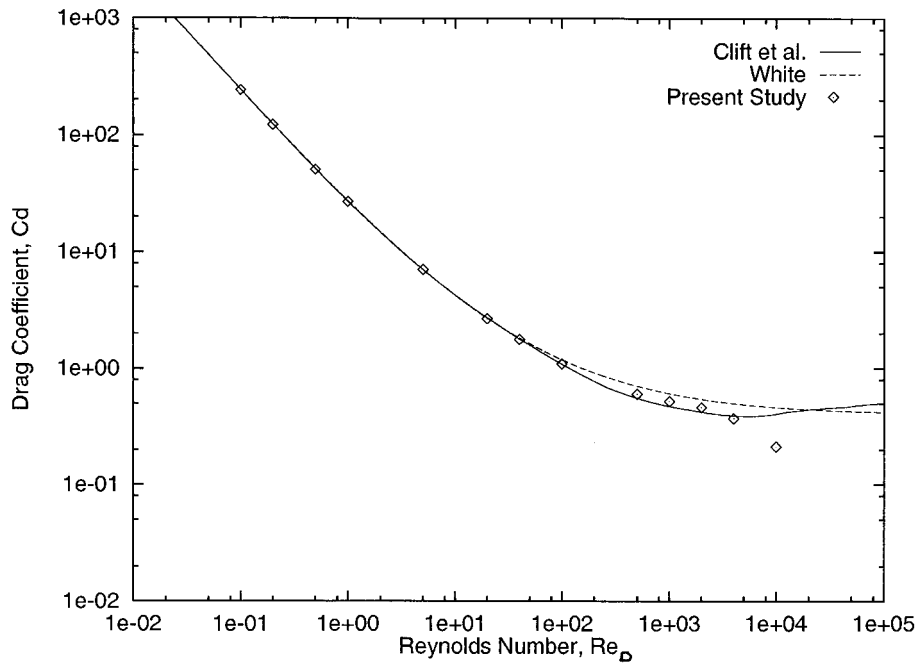


Fig. 1. Comparison of results for the steady-state drag coefficient with two correlations.

For the transient heat transfer computations, the dimensionless  $\Delta t$  was chosen to be in the range 0.001–0.05, depending on the value of the Peclet number (larger values of  $Pe$  correspond to smaller  $\Delta t$ ).

An analytic solution for the Nusselt number is known for this problem, valid for Stokesian flow and finite but low Peclet numbers [8,21]:

$$Nu = 2 \left[ 1 + Pe \left[ \frac{1}{2} \operatorname{erf} \left( Pe \frac{\sqrt{t}}{2} \right) + \frac{1}{Pe \sqrt{\pi t}} \exp \left( -\frac{Pe^2 t}{4} \right) \right] + \frac{1}{2} Pe^2 \ln Pe \right] \quad (26)$$

where  $t$  is made dimensionless by using the characteristic time for conduction  $\tau_s = \rho c \alpha^2 / k$  [23,24]. We have compared this analytical solution with the numerical results obtained for the case of Stokes' flow and of  $Pe = 0.25$ . The results of the comparison are shown in Fig. 4, where it is obvious that, at long times, the two solutions differ by less than 3%.

The effect of the Reynolds number of the flow on the transient Nusselt number for a given Peclet number is shown in Figs. 5–7. They correspond to  $Pe = 1, 10$  and 100, respectively. The range of  $Re_D$  in all the figures is from the very low values associated with Stokes' flow (creeping flow) to 2000. The Stokes' flow results were obtained by assuming a Stokesian velocity profile as in [9] for which the characteristic time is that

of conduction,  $\tau_s = \rho c \alpha^2 / k$ . It was observed that for  $Pe < 1$  the effect of  $Re_D$  on  $Nu$  is relatively low (the fractional difference between the curves is at most 9% in the case of Fig. 5). However, the effect of  $Re_D$  becomes significant when  $Pe$  increases. As shown in Figs. 6 and 7, where the maximum fractional difference between the curves for Stokesian flow and  $Re_D = 2000$  is more than 30% and more than 50%, respectively. This indicates that at intermediate or high values of  $Pe$ , one should not rely on results of studies where the Reynolds number is implicitly or explicitly assumed to be very small.

Oftentimes, the long-time asymptotic solution of a transient problem is of interest to the scientist or engineer. In the case of the problem of transient heat transfer from a sphere, Acrivos [22] obtained such a solution, valid for  $Pe > 5$ , assuming that the Reynolds number of the flow is very small. His expression is as follows:

$$Nu = 1.249 Pe^{1/3} + 0.922 \quad (27)$$

Table 3 shows the results obtained from the asymptotic Eqs. (26) and (27) as well as the numerical results obtained during this study. The results are given in terms of a Peclet number ( $Pe_D$ ) which is analogous to  $Re_D$  and is defined in terms of the particle diameter ( $Pe_D = Pr Re_D$ ).

There is no asymptotic value for  $Pe_D = 2$  because

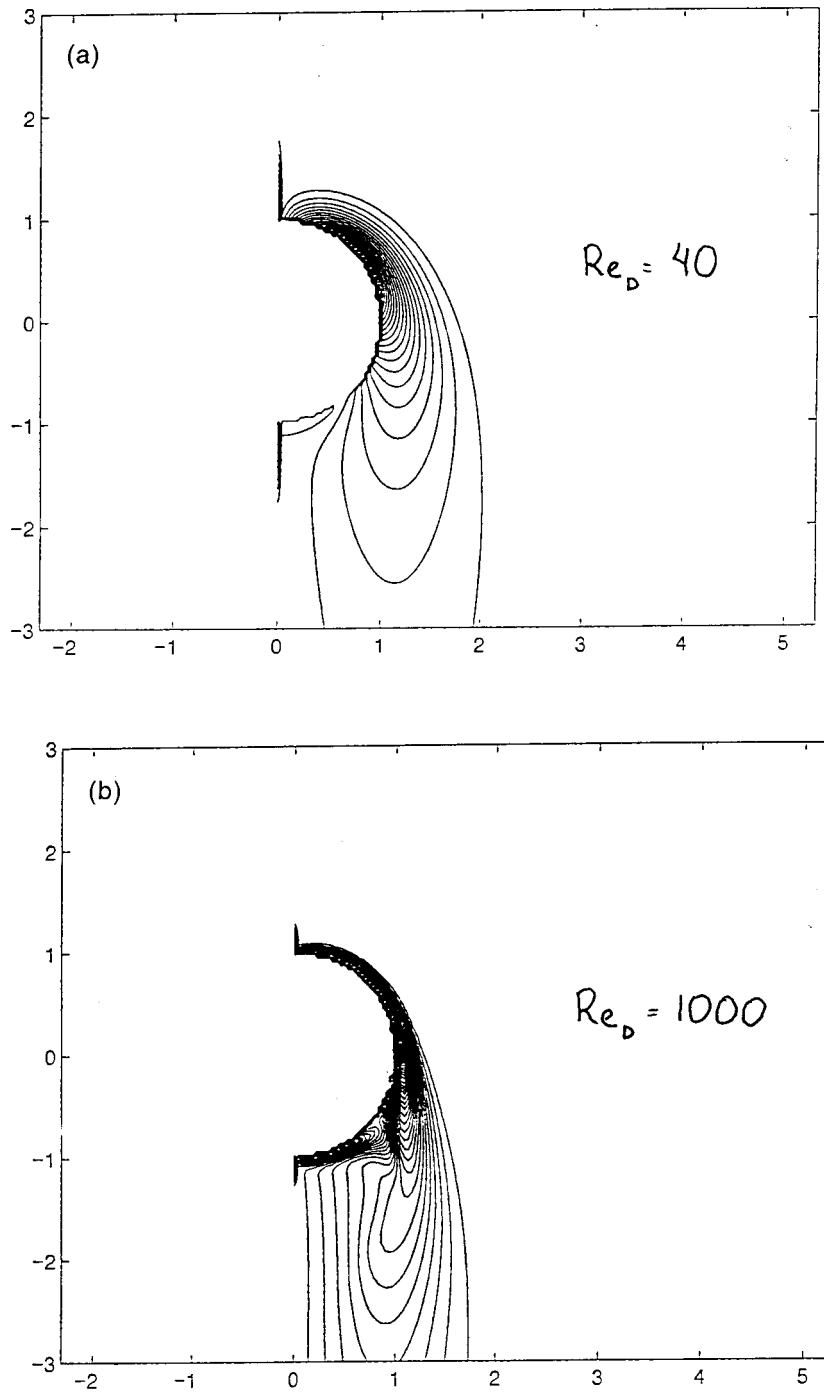


Fig. 2. Vorticity lines around the sphere for  $Re_D$  40 and 1000.

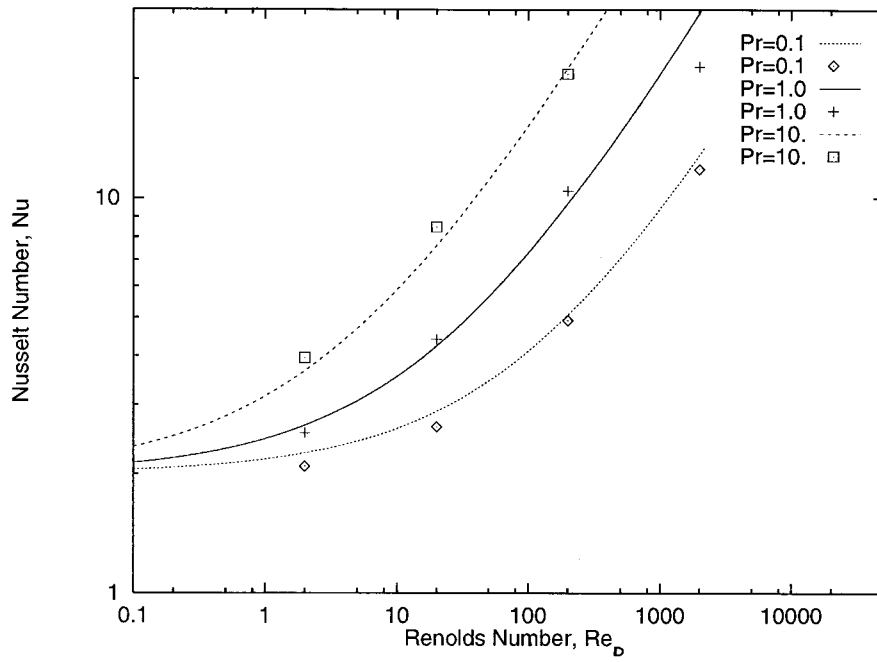


Fig. 3. Comparison of results for the steady-state Nusselt number with correlation.

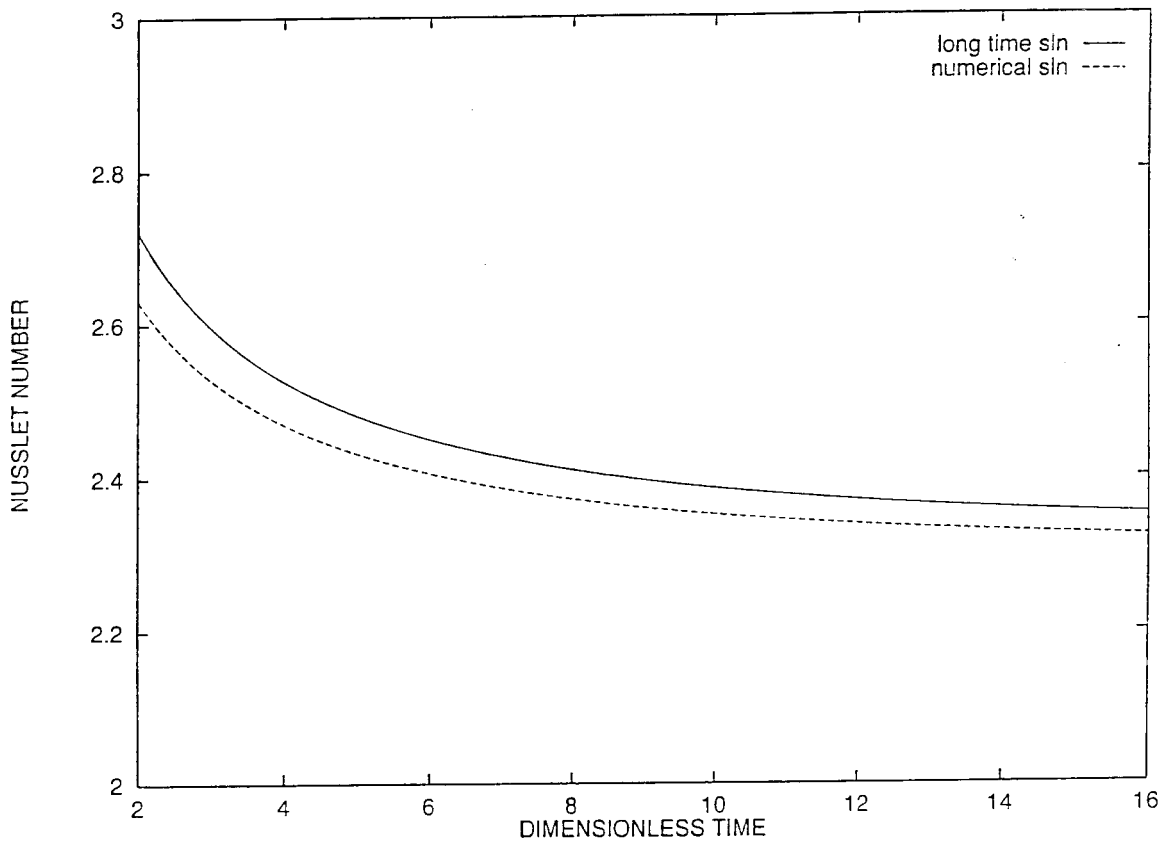


Fig. 4. Comparison of numerical and asymptotic results for Stokesian flow and  $Pe = 0.25$ .



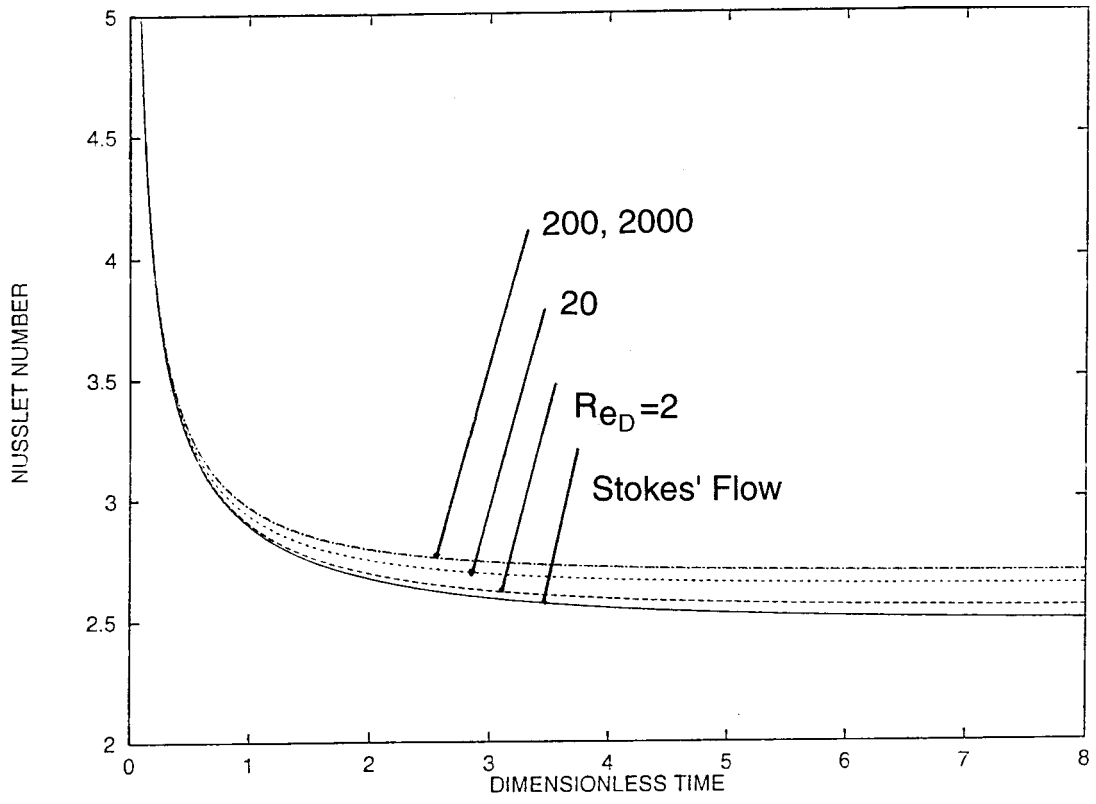


Fig. 5. The effect of  $Re_D$  on the transient  $Nu$ , for  $Pe = 1$ .

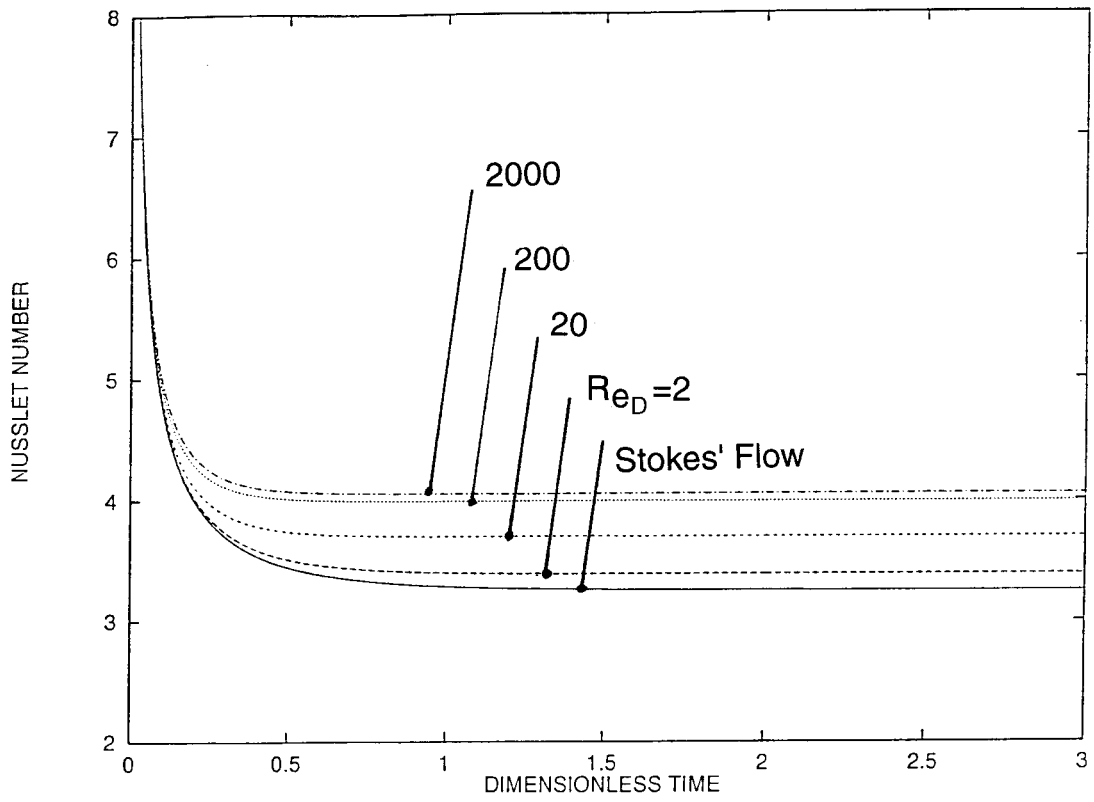


Fig. 6. The effect of  $Re_D$  on the transient  $Nu$ , for  $Pe = 10$ .

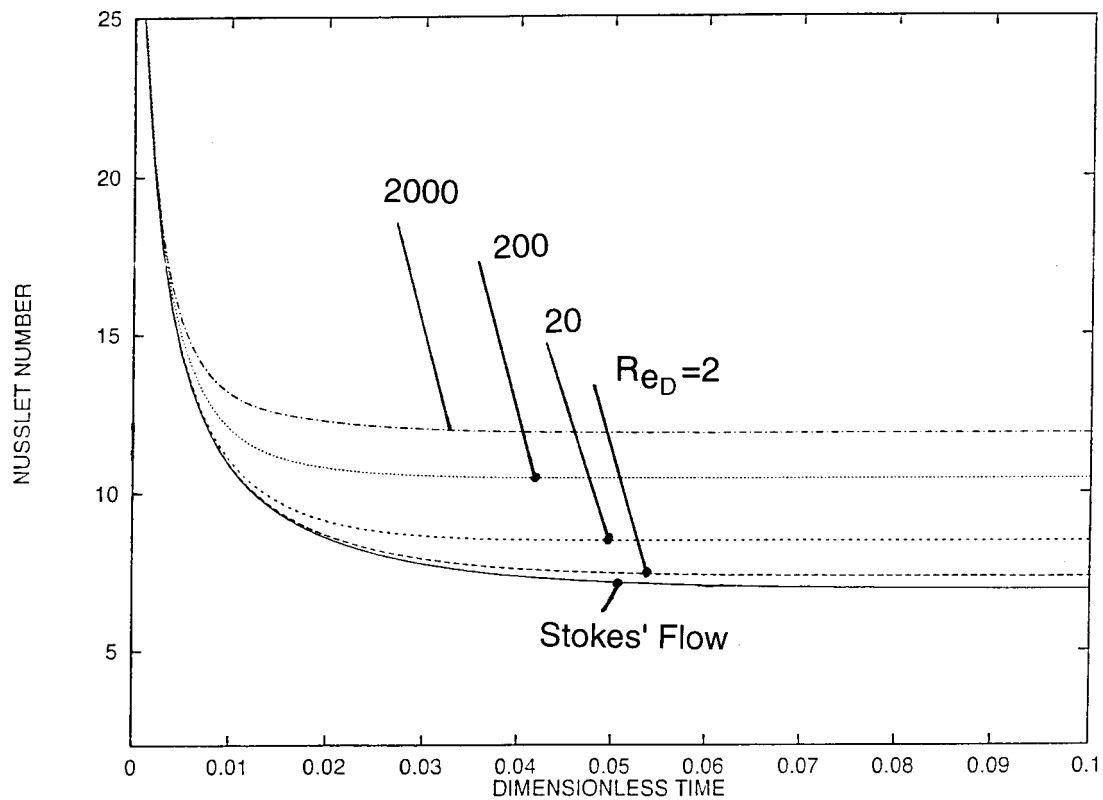


Fig. 7. The effect of  $Re_D$  on the transient  $Nu$ , for  $Pe = 100$ .

Eq. (26) is valid for  $Pe_D < 1$  and Eq. (27) for  $Pe_D > 10$ .

It is observed that the numerical solution agrees very well with the asymptotic steady-state solutions [8,21,22]. It is also observed that the effect of the particle Reynolds number on the steady-state heat transfer is insignificant when  $Pe$  is low, but that it becomes very important, when  $Pe$  is high. A simple correlation of these numerical data has been obtained. This correlation, which is reduced to the solution by Acrivos [22] at zero  $Re_D$  and also captures the dependence of the Nusselt number on the Reynolds and Peclet numbers, is as follows:

$$Nu = \left(0.922 + Pe_D^{1/3} + 0.1Re_D^{1/3}Pe_D^{1/3}\right) \quad (28)$$

## 5. Conclusions

The problem of the transient heat transfer from a sphere depends on both the velocity and the temperature field, which are developed in the vicinity of the sphere. Since the Reynolds number and the Peclet numbers are the parameters that determine these fields, the Nusselt number of the problem would depend on both of these parameters. The numerical method used

Table 3  
The effect of  $Re_D$  and  $Pe_D$  on the steady-state values of  $Nu$

$Re_D \backslash Pe_D$	0.2	0.5	1	2	10	20	100	200	1000	2000
Eq. (26) or (27)	2.077	2.163	2.327		3.06	3.61	5.52	6.72	10.84	13.41
0	2.086	2.180	2.303	2.487	3.25	3.77	5.68	6.92	11.24	13.91
2	2.093	2.200	2.338	2.545	3.38	3.95	6.01	7.34	11.95	14.74
20	2.098	2.217	2.382	2.639	3.70	4.40	6.89	8.47	13.75	16.70
200	2.099	2.224	2.402	2.692	3.99	4.91	8.27	10.44	17.31	20.63
2000	2.099	2.224	2.403	2.694	4.01	5.08	9.20	11.86	18.93	21.54

in this study enables one to perform calculations at relatively high Reynolds and Peclet numbers. The results of the study agree very well with previous numerical studies as well as with analytical and asymptotic solutions developed in the past. The results also show the dependence of  $Nu$  on both  $Re_D$  and  $Pe_D$ . It was found that when  $Pe_D < 2$  the dependence of the rate of heat transfer on the Peclet number is very weak. However, at higher values of  $Pe_D$ , both the transient rate of heat transfer and its asymptotic solution depend strongly on the value of  $Re_D$ .

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### References

- [1] J. Fourier, *Theorie Analytique de la Chaleur*, Paris, 1822.
- [2] A. Acrivos, T.E. Taylor, Heat and mass transfer from single spheres in Stokes flow, *Phys. of Fluids* 5 (1962) 387–394.
- [3] H. Brenner, Forced convection heat and mass transfer at small Peclet numbers from a particle of arbitrary shape, *Chem. Eng. Sci* 18 (1963) 109–115.
- [4] M. Kaviany, *Principles of Convective Heat Transfer*, Springer-Verlag, New York, 1994.
- [5] H.S. Carslaw, J.C. Jaeger, *Conduction of Heat in Solids*, Oxford University Press, Oxford, 1947.
- [6] P.N. Choudhury, D.G. Drake, Unsteady heat transfer from a sphere in a low Reynolds number flow, *Quart. J. Mech. and Appl. Math* 24 (1971) 23–36.
- [7] I. Proudman, J.R.A. Pearson, Expansions at small Reynolds numbers for the flow past a sphere and a circular cylinder, *J. Fluid Mech* 2 (1956) 237–262.
- [8] Z.-G. Feng, E.E. Michaelides, Unsteady heat transfer from a spherical particle at finite Peclet numbers, *J. Fluids Eng* 118 (1996) 96–102.
- [9] B. Abramzon, C. Elata, Heat transfer from a single sphere in Stokes flow, *Int. J. Heat and Mass Transfer* 27 (1984) 687–695.
- [10] H. Chiang, C. Kleinstreuer, Transient heat and mass transfer of interacting vaporizing droplets in a linear array, *Int. J. Heat and Mass Transfer* 35 (1992) 2675–2682.
- [11] C.H. Chiang, M.S. Raju, W.A. Sirignano, Numerical analysis of convecting, vaporizing, fuel droplets with variable properties, *Int. J. Heat Mass Transfer* 35 (1992) 1307–1324.
- [12] S.V. Patankar, *Numerical Heat Transfer and Fluid Flow*, Taylor and Francis, New York, 1980.
- [13] R. Courant, E. Isaacson, M. Rees, On the solution of non-linear hyperbolic differential equations by finite differences, *Com. Pure and Applied Math* 5 (1952) 243–256.
- [14] S.C.R. Dennis, J.D.A. Walker, Calculations of the steady flow past a sphere at low and moderate Reynolds numbers, *J. Fluid Mech* 48 (1971) 771–789.
- [15] B.P. LeClair, A.E. Hamielec, H.R. Pruppacher, A numerical study of the drag on a sphere at intermediate Reynolds and Peclet numbers, *J. Atmosph. Sci* 27 (1970) 308–315.
- [16] Z.-G. Feng, Heat transfer from small particles at low Reynolds numbers, Ph.D. dissertation, Tulane University, 1996.
- [17] F.M. White, *Viscous Fluid Flow*, McGraw-Hill, New York, 1991.
- [18] R. Clift, J.R. Grace, M.E. Weber, *Bubbles, Drops and Particles*, Academic Press, New York, 1978.
- [19] S. Whitaker, Forced convection heat transfer correlations for flow in pipes, past flat plates, single cylinders, single spheres, and flow in packed bed sand tube bundles, *AIChE J* 18 (1972) 361–371.
- [20] T. Yuge, Experiments on heat transfer from spheres including combined natural and forced convection, *Trans. ASME, J. Heat Transfer* 82 (1960) 163–175.
- [21] E.E. Michaelides, Z.-G. Feng, Analogies between the transient momentum and energy equations of particles, *Prog. Energy Comb. Sci* 22 (1996) 147–162.
- [22] A. Acrivos, A note on the rate of heat or mass transfer from a small particle freely suspended in linear shear field, *J. Fluid Mech* 98 (1980) 299–304.
- [23] X.Z. Din, E.E. Michaelides, Kinetic theory and molecular dynamics simulations of microscopic flows, *Phys. Fluids* 9 (1997) 3915–3925.
- [24] Z.G. Feng, E.E. Michaelides, Transient heat transfer from a particle with arbitrary shape and motion, *J. Heat Transfer* 120 (1998) 674–681.